

Math 101

First Exam
(24)

Name: _____

Number: _____

Section: 12:30
2:00

25/10/2008

Q1. Let $f(x) = x^2 + 4x$. If $g(x)$ is obtained by shifting $f(x)$ 2 units to the left and then 1 unit upward, then $g(x) = (x+2)^2 + 4(x+2) + 1$

Q2. Let $\text{Dom}(f) = [1, 7]$. If $g(x) = 2f(x-1)$, then $\text{Dom}(g) = [2, 8]$

Q3. If $f(x) = \frac{x+2}{x+1}$, then $\text{Dom}(f \circ f) = \mathbb{R} - \{1, 2\}$

Q4. $\cos^{-1}(\cos \frac{5\pi}{4}) = \frac{5\pi}{4}$

Q5. If $f(x) = \cos^2 x - \sin^2 x + 7$, then $\text{Range}(f) = [6, 8]$

Q6. If $f(x) = \ln x + \sqrt{2-x}$, then $\text{Dom}(f) = (0, 2]$

Q7. If $f(x) = x^3 + x + a$ is an odd function, then $a = 0$

$Q_1 = x^2 + 4x + (2)^2 = (x+2)^2 - 4$

$\therefore (x+2+2)^2 - 4 + 1 = (x+4)^2 - 3$

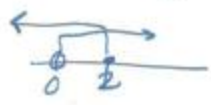
$Q_3 = \frac{\frac{x+2}{x+1} + 2}{\frac{x+2}{x+1} + 1} = \frac{R - \frac{1}{2}}{R - \frac{3}{2}}$

$\therefore D_{f \circ f} = \mathbb{R}$

Q6 $D \sqrt{2-x} = 2-x \geq 0$

$2 \geq x$

$(-\infty, 2]$



$x^3 + x + a$

1

$\frac{17}{225} \cdot 180 = 136$

$\frac{54180}{225} = 240.8$

$\cos(\pi - \pi) =$

$-\cos \alpha$

$-\cos 45$

$-\cos \frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

Q8. Solve $\log_2 x + \log_2(x-7) = \log_3 27$

$$\Rightarrow x^2 - 7x - 8 = 0$$

$$\log_2 x(x-7) = \log_3 27$$

$$\log_2(x^2 - 7x) = 3$$

$$1 = \frac{\ln(x^2 - 7x)}{\ln 2}$$

$$e^{\ln 2} = e^{\ln(x^2 - 7x)}$$

$$2 = (x^2 - 7x)$$

$$\Rightarrow x^2 - 7x - 2 = 0$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot (-2)}}{2}$$

$$= \frac{7 \pm \sqrt{49 + 8}}{2}$$

$$= \frac{7 \pm \sqrt{57}}{2} \quad x = \left(\frac{7 + \sqrt{57}}{2} \right)$$

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Q9. Let $f(x) = \frac{e^x - 1}{2e^x + 1}$. Find $f^{-1}(x)$.

$$\log_2(x^2 - 7x) = 3$$

$$3 \ln 2 = \ln(x^2 - 7x)$$

$$e^{3 \ln 2} = e^{\ln(x^2 - 7x)}$$

$$\frac{\ln(x^2 - 7x)}{\ln 2} = 3 \Rightarrow$$

$$8 = x^2 - 7x$$

$$x = \frac{e^y - 1}{2e^y + 1}$$

$$2x e^y + x = e^y - 1$$

$$2x e^y + e^y = -x - 1$$

$$e^y (2x + 1) = -x - 1$$

$$\ln e^y = \ln \frac{-x-1}{2x+1}$$

$$f^{-1}(x) = y = \ln \left(\frac{-x-1}{2x+1} \right) = \ln \left(\frac{1+x}{1-2x} \right)$$

~~$$3 \ln 2 = \ln(x^2 - 7x)$$~~

~~From~~

$$\Rightarrow x^2 - 7x - 8 = 0$$

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$$x = \frac{7 \pm \sqrt{49 - 4 \cdot 1 \cdot (-8)}}{2}$$

$$x = \frac{7 \pm \sqrt{81}}{2}$$

$$x = \frac{7 + 9}{2} = 8$$

$D \ln = (0, \infty)$