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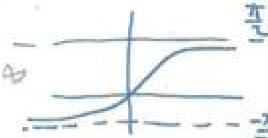
Math 101

NAME: ------ Num ---- Section: 12:30 → 2:00 29/11/2008

Q1. $\lim_{x\to 0} \dfrac{1-\cos 2x}{1-\cos 4x} = \dfrac{\sin 2x}{\ldots\ldots} = \frac{1}{2}$ $\frac{1}{4}$

Q2. $\lim_{x\to 1^+} e^{\frac{1}{1-x}} = e^{\frac{1}{1-1.1}} = e^{-10} = 0 = \frac{1}{e^{10}} = 0$

Q3. $\lim_{x\to\infty} \tan^{-1}(x^2 - x^4) = \ldots$ $-\frac{\pi}{2}$ $= \infty(-\infty) = -\infty$



Q4. If $\lim_{x\to 2} \dfrac{f(x)}{x-2} = 5$, then $\lim_{x\to 2} \dfrac{xf(x)}{x^2-4} = \lim_{x\to 2} \frac{x}{x+2} * \lim_{x\to 2} \frac{f(x)}{x-2} = \frac{2}{4} * 5 = \frac{1}{2} * 5 = \frac{5}{2}$

Q5. $\dfrac{d}{dx}(\sin^{-1}(e^{2x})) = \dfrac{1}{\sqrt{1-(e^{2x})^2}} * 2 * e^{2x}$

Q6. If $y = \frac{3}{2}x + 6$ is tangent to $y = c\sqrt{x}$ (curve) at $x = 4$, then $c = \frac{3}{2} = \frac{c}{2\sqrt{x}} \Rightarrow \frac{3}{2} = \frac{c}{2\sqrt{4}} \Rightarrow \boxed{c = 6}$

Q7. If $f(x) = \dfrac{1}{x^2 + x + a}$ has only one vertical asymptote, then $a =$ $\frac{1}{4}$

$x^2 + x + a = 0 \Rightarrow (x + \frac{1}{2})^2 = 0$

$\boxed{a = \frac{1}{4}}$ $x = -\frac{1}{2}$ V.A

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Q8. Find $\dfrac{d^{200}}{dx^{200}}\boxed{(2x^{100} + \sin x + 2^x)}$

$$\frac{d}{dx} = 200x^{99} + \cos x + 2^x \ln 2$$

$$\frac{d^2}{dx^2} = (200)(99)x^{98} - \sin x + (\ln 2)(\ln 2)2^x$$

$$\frac{d^3}{dx^3} = (19800)(98)x^{97} - \cos x + (\ln 2)^3 2^x$$

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$$\frac{d^4}{dx^4} = \boxed{(19800)(98)(97)x^{96} + \sin x + (\ln 2)^4 2^x}$$

$$\frac{d^{200}}{dx^{200}} = 0 + \sin x + (\ln 2)^{200} 2^x$$

Q9. Find all horizontal asymptotes of $f(x) = \frac{x+2}{\sqrt{9x^2+1}}$

$$\lim_{x\to+\infty} f(x) = \frac{x+2}{|x|\sqrt{9+\frac{1}{x^2}}} = \lim_{x\to+\infty} \frac{x+2}{x\sqrt{9+0}} = \frac{1}{(1)(3)} = \frac{1}{3} \Rightarrow y=\frac{1}{3} \boxed{H.A}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x+2}{-x\sqrt{9}} = \frac{1}{(-1)(3)} = \frac{-1}{3} \Rightarrow y = \frac{-1}{3} \boxed{H.A}$$

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Q10. Find the linear approximation of $f(x) = \sqrt[3]{x+1}$ at $a = 0$ and use it to estimate $\sqrt[3]{0.9}$

$$l(x) \approx f(a) + f'(a)(x-a)$$
$$\approx f(0) + f'(0)(x-0)$$
$$l(x) \approx 1 + \frac{1}{3}(x)$$
$$l(x) \approx 1 + \frac{1}{3}x$$

$f(0) = \sqrt[3]{1} = 1$

$f'(x) = \frac{1}{3}(x+1)^{-\frac{2}{3}}$

$f'(0) = \frac{1}{3}(1)^{-\frac{2}{3}} = \frac{1}{3}$

$f'(1) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(2)^2}} = \frac{1}{3}\frac{1}{2\sqrt[3]{2}}$

$\frac{1}{6\sqrt[3]{2}}$

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$$f(b) = f(a) + f'(a)(b-a)$$

$a = ~~0~~ ~~1~~ 1$

$b = ~~1~~ ~~0.9~~ 0.9$

$\sqrt[3]{-0.1+1} =$ ~~$1 + \frac{1}{6\sqrt[3]{2}}(0.9-1)$~~

$\sqrt[3]{0.9} =$ ~~[illegible]~~

$\sqrt[3]{\ }$

$x+1 = 0.9$

$x = 0.9 - 1$

$x = -0.1 = \frac{-1}{10}$

$$\sqrt[3]{-0.1+1} = 1 + \frac{1}{3}(-0.1)$$

$$\sqrt[3]{0.9} = 1 - \frac{1}{30} = \frac{30-1}{30} = \frac{29}{30} = 0.966$$

$-\frac{1}{3} * \frac{1}{10}$

.966

30) 290

270

200

180

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