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Evaluate the following integrals:

[1] $\int \frac{\sin^{-1}x}{x^2}dx = \int \underbrace{(\sin^{-1}x)}_{u}\underbrace{(x^{-2})dx}_{dv} = uv - \int v\,du$

$= (\sin^{-1}x)\left(\frac{x^{-1}}{-1}\right) - \int \left(\frac{-1}{x}\right)\left(\frac{1}{\sqrt{1-x^2}}\right)dx.$

$= -\frac{\sin^{-1}x}{x} + \int \frac{1}{x\sqrt{1-x^2}}dx$

$dx = \cos\theta\, d\theta$
$x = \sin\theta$
$x^2 = \sin^2\theta$
$1-x^2 = 1-\sin^2\theta = \cos^2\theta$
$\sqrt{1-x^2} = \sqrt{\cos^2\theta} = \cos\theta.$

$+ \int \frac{\cos\theta}{\sin\theta \cdot \cos\theta} d\theta$

$+ \int \csc\theta\, d\theta \cdot * \frac{\csc\theta + \cot\theta}{\csc\theta + \cot\theta}$

$\frac{1}{\sin\theta} + \frac{1}{\tan\theta}$

$* \int \frac{\csc^2\theta + \csc\theta\cot\theta}{\csc\theta + \cot\theta} d\theta = -\ln|\csc\theta + \cot\theta| + C$

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$= \frac{-\sin^{-1}x}{x} + \ln\left|\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right| + C$

[2] $\int \frac{dx}{2-\sin x+\cos x}$

let $\boxed{z = \tan\frac{x}{2}}$

$= \int \frac{\frac{1}{z^2+1}}{2 - \frac{2z}{z^2+1} + \frac{1-z^2}{z^2+1}} * = \int \frac{\frac{2}{1+z^2} *}{\frac{2z^2+2-2z+1-z^2}{z^2+1}}$

$2\tan^{-1}z = x$
$\frac{2}{1+z^2} = dx$

~~$\sin 2x = 2\sin x\cos x$~~
~~$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$~~

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$= \int \frac{z^2+1}{3+z^2-2z} * \frac{2}{1+z^2} dz$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$= \int \frac{2\,dz}{3+z^2-2z} = \int \frac{2\,dz}{z^2-2z+3} = 2\int \frac{1}{z^2-2z+1-1+3}$

$\Rightarrow \sin x = 2 * \frac{z}{\sqrt{z^2+1}} * \frac{1}{\sqrt{z^2+1}} = \frac{2z}{z^2+1}$

$= 2\int \frac{1}{(z-1)^2+2} dz =$

$z-1 = \sqrt{2}\tan\theta$
$dz = \sqrt{2}\sec^2\theta$
$(z-1)^2 + 2 = 2\sec^2\theta$

$\Rightarrow \cos x = \frac{1}{z^2+1} - \frac{z^2}{z^2+1} = \frac{1-z^2}{z^2+1}$

$2\int \frac{\sqrt{2}\sec^2\theta}{2\sec^2\theta} d\theta = \sqrt{2}\int \sec\theta\, d\theta$

$= \sqrt{2}\ln|\sec\theta + \tan\theta| = \sqrt{2}\ln\left|\frac{\sqrt{2+(z-1)^2}}{\sqrt{2}} + \frac{z-1}{\sqrt{2}}\right| = \sqrt{2}\ln$

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[3] $\int \frac{x^2+4x+8}{x^2(x^2+4)}dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$

$x^2+4x+8 = Ax(x^2+4) + B(x^2+4) + (Cx+D)(x^2)$

$x=0 \Rightarrow \frac{4B}{4} = \frac{8}{4} \Rightarrow \boxed{B=2}$

$x=1 \Rightarrow 13 = 5A + 10 + C + D$

$x=-1 \Rightarrow \begin{cases} 3 = 5A + C + D \\ 5 = -5A + 10 - C + D \end{cases}$

$8 = 10 + 2D \Rightarrow \frac{8-10}{2} = D \Rightarrow \boxed{D=-1}$

$3 = 5A + C - 1 \Rightarrow 4 = 5A + C \Rightarrow \cancel{C = 4 - 5A}$

$-5 = -5A - C - 1 \Rightarrow -4 = -5A - C \Rightarrow \cancel{4 - 5A - 4 + 5A}$

$\boxed{4 = 5A + C}$

$4 = 5A + C$

$8 = 10A + 2C$

$x=2 \Rightarrow 20 = 16A + 16 + 8C - 4$

$20 - 16 + 4 = 16A + 8C$

$\frac{8}{8} = \frac{16A}{8} + \frac{8C}{8}$

$1 = 2A + C \Rightarrow$

$-1 \times \quad 4 = 5A + C$

$\frac{-3}{-3} = \frac{-3A}{-3} \Rightarrow \boxed{A=1}$

$\boxed{C=-1}$

$\int \frac{1dx}{x} + \int \frac{2}{x^2}dx + \int \frac{-x-1}{x^2+4}dx$

$x^2+4=z$

$2x\,dx = dz$

$x\,dx = \frac{1}{2}dz$

$\ln|x| + \frac{-2}{x} + -\int \frac{x+1}{x^2+4}dx = \int \frac{x}{x^2+4} + \int \frac{1}{x^2+4}$

$= \frac{1}{2}\int \frac{dz}{z} + \frac{1}{2}\tan^{-1}\frac{x}{2}$

$\ln|x| - \frac{2}{x} - \frac{1}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

$x = 3\sec\theta$

$x^2 = 9\sec^2\theta$

$x^2 - 9 = 9\sec^2\theta - 9$

$\sqrt{x^2-9} = \sqrt{9\tan^2\theta} = 3\tan\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$x^3 = 27\sec^3\theta$

[4] $\int \frac{\sqrt{x^2-9}}{x^3}\,dx = \int \frac{3\tan\theta \cdot 3\sec\theta\tan\theta\,d\theta}{27\sec^3\theta}$

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$= \frac{9}{27}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{3}\int \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta\,d\theta$

$= \frac{1}{3}\int \sin^2\theta\,d\theta = \frac{1}{3}\int \frac{1-\cos 2\theta}{2}\,d\theta = \frac{1}{6}\int (1-\cos 2\theta)\,d\theta$

$= \frac{1}{6}\int 1-\cos 2\theta\,d\theta = \frac{1}{6}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$

$= \frac{1}{6}\left[\sec^{-1}\frac{x}{3} - \frac{2\sin\theta\cos\theta}{2}\right] = \frac{1}{6}\left[\sec^{-1}\frac{x}{3} - \frac{\sqrt{x^2-9}}{x} \times \frac{3}{x}\right] + C$

[5] Is the following improper integral convergent or divergent? If it is convergent, find its value. $\Rightarrow$ 1

$\int_0^1 \frac{e^{\frac{1}{x}}}{x^3}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{e^{\frac{1}{x}}}{x^3}\,dx = \int_t^1 e^{\frac{1}{x}} \cdot \frac{1}{x^2} \cdot \frac{1}{x}\,dx$

$\frac{1}{x} = z$

$-\frac{1}{x^2}\,dx = dz$

$= \lim_{t\to 0^+} -\int_t^1 e^z \cdot \frac{1}{x}\,z\,dz = \lim_{t\to 0^+} -\int_t^1 z e^z\,dz$

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$-\lim_{t\to 0^+} \left[z e^z - \int e^z\,dz\right]$

$-\lim_{t\to 0^+} \left[z e^z - e^z\right]_t^1 = \lim_{t\to 0^+} \left[e^z - z e^z\right]_t^1$

$= \lim_{t\to 0^+} [e - e] - [e^t - t e^t] = \lim_{t\to 0^+} [e^t] \quad$ zero $\quad -\lim e^t = \quad = -1$

$\therefore$ $\boxed{\text{converge} = -1}$

$\lim_{t\to 0^+} \; = \frac{e^t}{\frac{1}{t}} = \frac{e^t}{\frac{-1}{t^2}} = \frac{e^t}{\frac{1}{t^3}}$

$\Rightarrow \frac{t}{\frac{1}{e^t}} = \frac{1}{\frac{-e^t}{e^{t^2}}} = \frac{1}{\frac{-1}{e^t}} = 0 \quad \lim_{x\to 0^+} \frac{1}{\frac{-1}{e^t}} = \text{zero}$