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الفصل الصيفي	تفاضل وتكامل	الجامعة الأردنية
٢٠٠٨ – ٢٠٠٩	الاختبار الأول	قسم الرياضيات

اسم الطالب: [redacted]

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10:20
11:20

$20\frac{1}{2}$

Excellent

Complete the following statements(1.5 points each): point

[1] The equation of the line passing through (2,3,0) and perpendicular to both $2i+3k$ and $3i+4j+5k$ is

$v = (2i+3k)\times(3i+4j+5k) = \begin{vmatrix} i & j & k \\ 2 & 0 & 3 \\ 3 & 4 & 5 \end{vmatrix} \Rightarrow v = \langle -12, -1, 8\rangle$ ...... $x = 2 - 12t$, $y = 3 - t$, $z = 8t$

[2] A vector that has the same direction as $\langle -2,3,6\rangle$ but has length 4 is

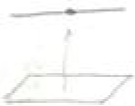
$\vec{v} = \frac{4}{7}\vec{w}$

$\vec{v} = \frac{4}{7}\vec{w} \Rightarrow |v| = |\frac{4}{7}||w|$

$|v| = \frac{4}{7}\cdot 7 \Rightarrow \boxed{|v| = 4}$

[3] The distance between the line $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z+3}{-3}$ and the plane $x+2y+2z=13$ is $\frac{20}{3}$ ...... $= \frac{|1-2-6-13|}{\sqrt{1+4+4}}$ = ...

they are parallel



[4] The equation $4x^2 - y^2 - 2z^2 = -16$ is an equation of $\Rightarrow -\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{8} = 1$ hyperboloid of one sheet ... which is symmetric around ..x.. axis.

[5] The area of the triangle whose vertices are A(2,1,5), B(−1,3,4) and C(3,0,6) is $\frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{\sqrt{6}}{2}$ ......

[6] The projection of $2i-3j+5k$ along $i+2j+6k$ is $proj_a b = \frac{a\cdot b}{|a|^2}\cdot a$

$= \frac{26}{41}\langle 1,2,6\rangle = \langle \frac{26}{41}, \frac{52}{41}, \frac{156}{41}\rangle$ ......

[7] Let $\bar{a} = \langle 4, 0, -3\rangle$. Then the direction angle $\gamma = \cos\gamma = \frac{-3}{5}$

$\cos\alpha = \frac{4}{5}$

$\cos\beta = 0$

Answer the following questions(Show your work)

[8] Consider the two planes $P_1: x+y+z=1$ and $P_2: x+y=2$

(a) (3 points) Find the line of intersection between P_1 and P_2.

~~$z = t$~~
~~$x + y = 1 - t$~~
~~$x + y = 2$~~

let $\boxed{y = 0}$

$\Rightarrow \boxed{x = 2}$

$\boxed{z = -1}$

the point is $(2, 0, -1)$

the vector is $\langle -1, 1, 0\rangle$

$\vec{v} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = i(0-1) - j(0-1) + k(1-1)$

$= -\hat{i} + \hat{j} = \langle -1, 1, 0\rangle$

$\boxed{x = 2 - t}$ $\boxed{y = t}$ $\boxed{z = -1}$

(b) (2 points) Find the acute angle between P_1 and P_2.

$n_1 = \langle 1, 1, 1 \rangle$

$n_2 = \langle 1, 1, 0 \rangle$

$\cos\theta = \dfrac{n_1 \cdot n_2}{|n_1||n_2|} \Rightarrow \cos\theta = \dfrac{1+1+0}{\sqrt{3}\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{2}{\sqrt{6}} \Rightarrow \theta = \cos^{-1}\dfrac{2}{\sqrt{6}}$

[9] Consider the two lines

L_1: $x = -1 + t$; $y = 2 + t$; $z = 1 - t$ and

L_2: $x = 1 - 4s$; $y = 1 + 2s$; $z = 2 - 2s$.

(a) (2 points) Find the point of intersection of the two lines.

$-1 + t = 1 - 4s \Rightarrow t + 4s = 2$

$2 + t = 1 + 2s \Rightarrow -t + 2s = -1$

$6s = 3 \Rightarrow \boxed{s = \tfrac{1}{2}}$

$\Rightarrow \boxed{t = 0}$

$\boxed{z_1 = 1}$

$z_2 = \boxed{z_2 = 1}$

the two lines are intersects and the point of intersecting is

$P = (-1, 2, 1)$

(b)(3 points) Find an equation of the plane determined by L_1 and L_2.

$P = (-1, 2, 1)$

$n = r_1 \times r_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -4 & 2 & -2 \end{vmatrix} = i(-2+2) - j(-2-4) + k(2+4)$

$= -2j + 6k = \langle 0, +6, 6 \rangle$

$\cancel{0(x+1) - 2(y-2) + 6(z-1) = 0}$

$\cancel{-2y + 4 + 6z - 6 = 0}$

$\cancel{-2y + 6z = 2} \Rightarrow \cancel{-y + 3z = 1}$

$0(x+1) + 6(y-2) + 6(z-1) = 0$

$6y - 12 + 6z - 6 = 0$

$6y + 6z = 18$

$\boxed{y + z = 3}$