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Complete the statements 1 – 10. (2 points each) $\Rightarrow t > -1$

$\Rightarrow \mathbb{R} - \{0\}$ $t+1 > 0 \Rightarrow$

[1] The domain of $\bar{r}(t) = \left\langle \frac{e^{-t}-1}{t}, \frac{1}{\sqrt{t+1}} \right\rangle$ is $(-1, \infty) - \{0\}$

[2] The equation of the normal plane to the curve $t = \pi$

$x = 2\sin t$, $y = t$, $z = 2\cos t$ at the point $(0, \pi, -2)$ is $\langle -2, 1, 0 \rangle \Rightarrow$ $x = -2t$, $y = \pi + t$, $z = -2$ $\quad \frac{x}{-2} = y - \pi = 0$

tangent plane $\Rightarrow$ $\boxed{y - 2x = \pi}$

[3] If the directional derivative

$D_{\bar{a}} f(1,2,-1) = -3$, $|\nabla f(1,2,-1)| = 3$ and $\bar{a} = \langle 1,2,-5 \rangle$, then $\nabla f(1,2,-1) =$

$\left\langle \frac{-3}{\sqrt{30}}, \frac{-6}{\sqrt{30}}, \frac{+15}{\sqrt{30}} \right\rangle$

[4] If $\bar{r}(t) = \langle e^{2t}\cos 2t, e^{2t}\sin 2t, 2 \rangle$, then $\bar{T}(0) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$

[5] $\lim\limits_{(x,y)\to(0,0)} \frac{3xy^4}{x^2+y^8}$ along the path $x = y^4$ is $\frac{3}{2}$

−4

[6] If $f(r,\theta) = \theta\cos(r+\theta)$, then $f_{r\theta} = -\sin(r+\theta) - \theta\cos(r+\theta)$

$f(0,0) + f_x(x-0) + f_y(y-0)$

[7] The local linear approximation of $f(x,y) = \sqrt{y + \cos^2 x}$ at $(0,0)$ is

$\boxed{1 + \frac{y}{2}}$

[8] If $xyz = \cos(x+y+z)$, then $\frac{\partial z}{\partial y} = \frac{-f_y}{f_z} = -\left[\frac{-\sin(x+y+z) - xz}{-\sin(x+y+z) - xy}\right]$

[9] The maximum rate of change of $f(x,y,z) = \frac{x+y}{z}$ at $(1,1,-1)$ is

$|\nabla f(1,1,-1)| = |\langle -1, -1, -2 \rangle| = \sqrt{(-1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

[10] Let $w = \sqrt{x+y}$, $x = r\cos\theta$, $y = r\sin\theta$. Then $\frac{\partial w}{\partial r} = \frac{dw}{dx}\frac{dx}{dr} + \frac{dw}{dy}\frac{dy}{dr}$

$\left(\frac{1}{2\sqrt{x+y}}\right)(\cos\theta) + \left(\frac{1}{2\sqrt{x+y}}\right)(\sin\theta) = \frac{\cos\theta}{2\sqrt{x+y}} + \frac{\sin\theta}{2\sqrt{x+y}}$

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[11] If the length of the diagonal of a rectangular box must be $\boxed{\sqrt{12}\ cm}$, what is the largest possible volume? (Hint: The diagonal of a rectangular box with adjacent edges x,y and z is $\sqrt{x^2+y^2+z^2}$)(3 points).

① $D = \sqrt{x^2+y^2+z^2} - \sqrt{12} = 0 \Leftarrow g(x,y,z)$

② $V = xyz \Rightarrow f(x,y,z)$

$\Rightarrow \nabla f(x,y,z) = \lambda g(x,y,z) \Rightarrow$

$f_x = yz = \lambda \frac{x}{\sqrt{x^2+y^2+z^2}}$

$f_y = xz = \lambda \frac{y}{\sqrt{x^2+y^2+z^2}}$

$f_z = xy - \lambda \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\Rightarrow x \neq 0, y \neq 0, z \neq 0$

$\frac{yz}{x}\cancel{\sqrt{x^2+y^2+z^2}} = \frac{xz}{y}\cancel{\sqrt{x^2+y^2+z^2}} \Rightarrow x^2\cancel{z} = y^2\cancel{z}$ $\boxed{x = y}$

$\Rightarrow \frac{xz}{z} * \cancel{\sqrt{x^2+y^2+z^2}} = \frac{xz}{x}\cancel{\sqrt{x^2+y^2+z^2}} \Rightarrow \boxed{x^2 = z^2}$ $\boxed{x = z = y}$

$\sqrt{3}\, x = \sqrt{12} \Rightarrow \boxed{x = y = z = \sqrt{4} \mp 2}$

[12] Let $f(x,y) = e^y(y^2 - x^2)$. Find the point at which f has a maximum value. (5 points) $e^y y^2 - e^y x^2$

$f_x = -2xe^y = 0 \Rightarrow \boxed{x = 0}$

$f_y = e^y y^2 + 2ye^y - e^y x^2 = 0$

$e^y[y^2 + 2y - x^2] = 0 \Rightarrow y^2 + 2y - x^2 = 0$

(0,0)

$D = f_{xx} f_{yy} - (f_{xy})^2$

$(-2)(2) - (0) = -4 \Rightarrow$ saddle point (0,0)

$\boxed{f \Rightarrow \text{don't hav max or min value}}$

-3

(0,-2) ⇒

$D = (-2)\left(\frac{-2}{e^2}\right) - (0) = \frac{4}{e^2}$ > zero $f_{xx} < 0$ relative max

[13] Find the curvature of $\vec{r}(t) = \langle \cos t, 2\sin t \rangle$. (2 points)

$r' = \langle -\sin t, 2\cos t \rangle$

$r'' = \langle -\cos t, -2\sin t \rangle$

$k = \frac{|r' \times r''|}{|r'|^3} = \frac{|2\sin^2 t + 2\cos^2 t|}{\left(\sqrt{1 + 3\cos^2 t}\right)^3}$

$\begin{vmatrix} -\sin t & 2\cos t & 0 \\ -\cos t & -2\sin t & 0 \end{vmatrix}$

$= \frac{2}{(1+3\cos^2 t)^{3/2}}$

$|r' \times r''| = 2\sin^2 t + 2\cos^2 t$

$\sqrt{\sin^2 t + 4\cos^2 t}$

$\sqrt{1 - \cos^2 t + 4\cos^2 t}$