

note: the final Exam consists of two part; part 16 NCQ, choose and circle the best and right answer parts II, 4 problems, solve.

Part I: 32 points

1- one of the polar coordinates of the point

$$P(x, y) = (-\sqrt{3}, 1) \text{ is:}$$

$$\left(2, \frac{5\pi}{6}\right)$$

$$\text{b) } \left(2, -\frac{\pi}{6}\right)$$

$$\left(-2, \frac{\pi}{6}\right)$$

$$\text{d) } \left(-2, -\frac{5\pi}{6}\right)$$

By transforming from rectangular to polar coordinates, the equation $x^2 + y^2 - 6y = 0$ becomes:

a) $r = 3 \sin \theta$

b) $r = 3 \cos \theta$

c) $r = 6 \sin \theta$

d) $r = 6 \cos \theta$

3- By transforming from polar to rectangular coordinates, the equation $r^2 \cos 2\theta = 9$ becomes;

a) $xy = 9$

b) $x^2 - y^2 = 9$

c) $x + y = 9$

d) $y^2 - x^2 = 9$

4- the length of the curve $y = \sqrt{1 - x^2}, -1 \leq x \leq 1$ is

a) π

b) $\frac{\pi}{2}$

c) 2π

d) $\frac{\pi}{4}$

5) the volume of the solid generated by rotating the region under the curve $y = e^{-x^2}$ over $1 \leq x < \infty$ around the y-axis is:

- a) Π b) $\frac{\Pi}{e}$ c) $\frac{1}{e}$ d) $\frac{e}{\Pi}$

6) $\int_1^4 \frac{\ln x}{\sqrt{x}} dx =$

- a) $2 \ln 2$ b) $4 \ln 2 - 8$
 c) $8 \ln 2 - 4$ d) $2 \ln -4$

7) the eccentricity e of the ellipse $25x^2 + 9y^2 = 225$ is

- a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{5}{4}$ d) $\frac{5}{3}$

8) the equation of the ellipse whose eccentricity $e = \frac{2}{3}$, focus at $(0,0)$, and the corresponding directrix is $x=2$ is:

- a) $x^2 + 3y^2 + 8x - 8 = 0$ b) $3x^2 + 8y^2 - 8 = 0$
 c) $5x^2 + 8y^2 + 16x - 16 = 0$ d) $9x^2 + 5y^2 + 16y - 16 = 0$

9) the equation of the hyperbola whose asymptotes are: $y = \mp \frac{12}{5}x$ and vertices $(0, \mp 24)$ one:

- a) $\frac{x^2}{25} - \frac{y^2}{144} = 1$ b) $\frac{y^2}{144} - \frac{x^2}{25} = 1$
 c) $\frac{x^2}{100} - \frac{y^2}{576} = 1$ d) $\frac{y^2}{576} - \frac{x^2}{100} = 1$

10) Let $x'y'$ be the coordinate axes obtained by rotating the xy -axes to eliminate the xy term.

If the coordinates of the focus of the parabola

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$$

In the $x'y'$ coordinate system are $(0, \frac{1}{4})$, then the coordinates of the focus in the xy system are:

- a) $(-\frac{\sqrt{3}}{8}, \frac{1}{8})$ b) $(-\frac{1}{8}, \frac{\sqrt{3}}{8})$

$$\left(\frac{1}{8}, \frac{\sqrt{3}}{8}\right)$$

$$\text{d) } \left(\frac{\sqrt{3}}{8}, -\frac{1}{8}\right)$$

$$11) \lim_{n \rightarrow \infty} \text{Lim} \left(\frac{n+3}{n+1}\right)^{n+2}$$

$$\text{a) } e^2$$

$$\text{b) } e^{-2}$$

$$\text{c) } e^{\frac{1}{2}}$$

$$\text{d) } e^{-\frac{1}{2}}$$

12) the series

$$\frac{1}{2} - \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \frac{1}{32} - \frac{1}{64} - \frac{1}{128} + \dots \text{is}$$

a) conditionally convergent

b) absolutely convergent

c) Neither absolutely nor conditionally convergent.

d) divergent

13) the series $\sum_{k=1}^{\infty} \frac{1}{1 + \left(\frac{1}{2}\right)^k}$: is

- a) convergent by comparison with $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$
- b) convergent because it is geometric and $r = \frac{1}{2}$
- c) Divergent by the integral test.
- d) Divergent by the divergence test.

14) the sin of the series $\sum_{n=0}^{\infty} \left(\frac{3}{2^n} - \frac{2}{3^n}\right)$ is:

$$\text{a) } -2 < x < 2$$

$$\text{b) } -2 \leq x \leq 2$$

$$\text{c) } -1 < x < 1$$

$$\text{d) } -1 \leq x \leq 1$$

16) the Taylor series of $\sin x$ at $-1 \leq x \leq 1$

$$\text{a) } \sum_{n=0}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2}\right)^{2n+1}}{(2n+1)!} \quad \forall x.$$

$$\text{b) } \sum_{n=0}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!} \quad \forall x.$$

$$\text{c) } \sum_{n=0}^{\infty} \frac{\left(x - \frac{\pi}{2}\right)^{2n+1}}{(2n+1)!} \quad \forall x.$$

$$d) \sum_{n=0}^{\infty} \frac{\left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!} \forall x.$$

End of Part I

Go to Part II

بسم الرحمن الرحيم

الخميس 98/8/13

الامتحان النهائي

الجامعة الأردنية

Math 102

قسم الرياضيات

اسم الطالب:

رقم الطالب الجامعي:

Q₁: the parametric equations of the circle $(x+1)^2 + (y-2)^2 = 1$ in clockwise direction are:

a) $X=1-\cos t$, $y=2-\sin t$	$0 \leq t \leq 2\pi$
b) $X=-1+\sin t$, $y=2+\cos t$	$0 \leq t \leq 2\pi$
c) $x=1+\sin t$, $y=2-\cos t$	$0 \leq t \leq 2\pi$
d) $x=-1+\cos t$, $y=2+\sin t$	$0 \leq t \leq 2\pi$

Q₂ the equation of the tangent line to the curve C:

$$x = -\sqrt{2} \cos t \quad \leq t \leq 2\pi$$

$$y = 2\sqrt{2} \sin t$$

At (1,2) is

a) $y-x=1$

b) $y+2x=4$

c) $y+x=3$

d) $2y+x=5$

Q₃: the equation for the conic section with vertices (1,0), (-5, π)

and the pole at a focus is

$$r = \frac{5}{2-5\cos\theta}$$

b) $r = \frac{5}{2-5\sin\theta}$

$$r = \frac{5}{2+3\sin\theta}$$

d) $r = \frac{5}{2+3\cos\theta}$

Q₄: $\sum_{n=1}^{\infty} (-1)^n \frac{(\ln 3)^n}{i^n}$ is

a) \ln_3

b) 2

c) $-\frac{2}{3}$

e) 3

Q₅: the radw of cenvrgence of $\sum_{n=1}^{\infty} n! \frac{(3x)^n}{n^n}$ is
 a) $\frac{e}{3}$ b) 3 ero c) $\frac{e}{2}$ d) ∞

Q₆: of the area of the regson enelsed by the rose curve $r = \sin(88)$ is A, then then the area enclosed by $r = 3 \sin(88)$ is
 a) 3A b) $\frac{A}{3}$ c) qA d) A^3

Q₇: $\int_2^3 x^3(4-x^2)^3 dx =$

- $-\frac{3}{5}$ b) $+\frac{3}{5}$
 $\frac{25}{2} \left(\frac{1}{\sqrt{5}} - 1 \right)$ d) $\frac{25}{2} \left(\frac{1}{\sqrt{5}} + 1 \right)$

Q₈: the graph of $r^{-1} = 3 + 3 \cos \theta$ is

- a) Hyperbola with $e=3$ b) cardioid
 c) parabola d) circle

Q₉ : the polar equation $r^2 =$ is symmeric abeut

- a) only the (pole) b) the y-axis (only)
 c) All the axes and pole d) the x-axis only

Q₁₀ : the polar equation of the cure $(x^2 + y^2)^3 = L_1 x^2 y^2$

- $r^3 = \sin 2\theta$ b) $r^2 = 4 \sin^2 2\theta$
 $r = \sin 2\theta$ d) $r^2 = 2 \cos 2\theta$

Q₁₁: what is the length from $x=0$ to $\frac{\Pi}{2}$ of the curve whose slope in this interval is $\frac{dy}{dx} = \sqrt{\cos x}$

- a) $2\sqrt{2}$ b) 1 c) 2Π d) 2

Q₁₂: the area inside the cardioid $r = 1 - \cos \theta$ and outside the circle $r = 1$ is

- a) $4 - \frac{\pi}{2}$ b) $4 + \frac{\pi}{2}$ c) $1 + \frac{\pi}{8}$ d) $2 + \frac{\pi}{4}$

Q₁₃: the base of a solid S is rectangle bounded by $X = 0$, $X = 8$, $y = 3$, $y = -3$ and of

Every cross section perpendicular to the x-axis is a semicircle with its diameter in the xy - planes, then the Volume of S is

- a) 36π b) 27π c) 72π d) 54π

Q₁₄: $\lim_{n \rightarrow \infty} \frac{(-1)^n \cdot n}{n^3}$ if $a_n = 0$ $0 < n < \infty$

Then $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{n^3}$ is

- a) diverges b) Absolutely convergent
 b) conditionally convergent d) we can't decide.

Q₁₅: consider the region R in the figure, then the volume of the solid generated by revolving R about the line $x = 4$, Using the shell method is given by:

- a) $2\pi \int_0^4 (2 - \sqrt{x})x \, dx$
 b) $2\pi \int_0^2 (2 - x)(4 - x^2) \, dx$
 c) $2\pi \int_0^2 (4 - x^2) \, dx$
 d) $2\pi \int_0^4 (4 - x)(2 - \sqrt{x}) \, dx$

Q₁₆: $\int_1^4 \sqrt{x} \cdot \ln x \, dx =$

- a) $\frac{32 \ln 2}{3} - \frac{28}{9}$ b) $\frac{16 \ln 2}{2} - \frac{4}{9}$

$$\frac{16 \ln 2}{3} - \frac{4}{3}$$

$$\text{d) } \frac{32 \ln 2}{3} - 4$$

$$\text{Q}_{17}: \int_0^4 \cos \theta \sin \theta \, d\theta =$$

$$\frac{2\sqrt{21}}{3\sqrt{2}}$$

$$\text{b) } \frac{8-3\sqrt{3}}{12}$$

$$\frac{-2}{3}$$

$$\text{d) } 1$$

$$\text{Q}_{18}: \sum_{n=1}^{\infty} (-1)^{2n} + (-1)^{2+1} i^n$$

$$\text{a) } -1$$

$$\text{b) } 1$$

$$\text{c) } 0$$

$$\text{d) } \text{diverges.}$$

$$\text{Q}_{19}: \int_{-1}^0 x \sqrt{x+1} \, dx$$

$$\text{a) } \frac{2}{3}$$

$$\text{b) } -\frac{1}{3}$$

$$\text{c) } -\frac{4}{15}$$

$$\text{d) } 0$$

$$\text{Q}_{20}: \int_{\frac{\pi}{3}}^{\frac{4}{3}} \frac{dx}{1 - \sin x} =$$

$$\text{a) } \frac{1}{2} - \frac{1}{3}$$

$$\text{b) } \frac{1}{2} + \frac{1}{3}$$

$$\text{c) } \frac{1}{3} - \frac{1}{2}$$

$$\text{d) } \frac{1}{3} + \frac{1}{2}$$

$$\text{Q}_{21}; \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+x+1} \text{ is a possible partial fraction of:}$$

$$\frac{x+3}{(x-1)^2(x^3+1)}$$

$$\text{b) } \frac{x+3}{(x-1)^2(x^3+1)}$$

$$\frac{5x+3}{(x-1)^3(x^3-1)}$$

$$\text{d) } \text{none}$$

of the above

$$\text{Q}_{22}: \text{let } r=2-2 \sin \theta, \text{ then } \frac{dx}{d\theta} = 0 \text{ when } \theta$$

$$\left\{ \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$\left\{ \frac{\pi}{2}, \frac{-\pi}{6}, \frac{-5\pi}{6} \right\}$$

b

$$\left. \begin{array}{l}) \\ \left\{ \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\} \\) \end{array} \right\}$$

$$d \left\{ \frac{\pi}{2}, \frac{-\pi}{6}, \frac{-5\pi}{6} \right\}$$