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University of Jordan — Engineering Mathematics — 2^{nd} Exam
Math. Dept. — Math 202 — 16/12/2004
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Q1) Fin in the blanks with the final answer. **(9 points)**

1) Let $y'' + 4y = 8x^2$. Then the general form of a particular solution (do not evaluate the coefficients) is : $K_2 x^2 + K_1 x + K_0$

2) Let $y'' + 2y' + y = e^{-x}$. Then the general form of a particular solution (do not evaluate the coefficients) is : $C x^2 e^{-x}$

3) Let $y'' + 2y' + 5y = 40\cos(4x)$. Then the general form of a particular solution (do not evaluate the coefficients) is : ~~$A\cos$~~ $A\cos 5x + B\sin 5x$

4) Let $y'' + P(x)y' + Q(x)y = \sec(x)$. It is known that $y_1 = \cos(x)$ and $y_2 = \sin(x)$ are solutions of the homogeneous part of the D.E. Then a particular solution

$W = 1$

is $-\cos\int \sin x \sec x\, dx + \sin x \int \cos x \sec x = +\cos x \ln|\cos x| + x\sin x$

5) Let $y''' - 2y'' - y' + 2y = 0$. Then its solution is $c_1 e^{2x} + c_2 e^{-x} + c_3 e^{x}$

6) Let $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln(x)$. It is known that $y_1 = x$, $y_2 = x^2$, $y_3 = x^3$ are solutions of the homogeneous part. Then a particular solution

is $y_1 \int \frac{W_1(x)\, x\ln x}{W(x)} + y_2 \int \frac{W_2(x)\, x\ln x}{W(x)} + y_3 \int \frac{W_3(x)\, x\ln x}{W(x)} =$

$x \int \frac{x^4\, x\ln x}{2x^3} + x^2 \int -\frac{2x^3\, x\ln x}{2x^3} + x^3 \int \frac{x^2\, x\ln x}{2x^3}$

7) The D.E. $x^3 y'' + x(4x-1)y' + 2y = 0$ has a singular point at $x = 0$.

8) The indicial equation of : $x^2 y'' + xy' + y = 0$ is : $(r)(r-1) + r + 1 = 0$

$r^2 + 1 = 0$

9) Some one was trying to obtain the series solution of $y'' + xy' + y = 0$ and he obtained that

$$\sum_{k=0}^{\infty} \left[(k+2)(k+1) a_{k+2} + (k+1) a_k \right] x^k = 0$$

Then the general formula for a_{2k} is : $\frac{x^{2n}}{(2n)!}$ for $k \geq 1$.

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$a_{2k} =$

$(2k+2)$ ~~a_{2k}~~ $+ 2$.

$(2k+2)\, a_{2k+2}$

$=$ ~~$\frac{x^{2n}}{(2n)!}$~~ $\rightarrow \frac{(-1)\, x^{2n}}{2n!}$

Q2) Given the differential equation:

$$y''' + 5y'' + 7y' + 3y = 5e^{-x} + x^2 + 2\sin x$$

a) Find the solution of the corresponding homogeneous equation of the given equation.

b) write a suitable particular solution of the given equation by using the undetermined coefficients method (Do not evaluate the coefficients)

$$r^3 + 5r^2 + 7r + 3 = 0$$

$$
\begin{array}{r|l}
 & r^2 + 4r + 3 \\
r+1 & r^3 + 5r^2 + 7r + 3 \\
 & -r^3 - r^2 \\
 & 4r^2 + 7r + 3 \\
 & -4r^2 - 4r \\
 & 3r + 3 \\
 & -3r - 3
\end{array}
$$

$= e^{-x}$

$y' = -e^{-x}$

$y'' = e^{-x}$

$y''' = -e^{-x}$

$-e^{-x} + 5e^{-x} - 7e^{-x} + 3e^{-x}$

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$$(r+1)(r^2 + 4r + 3) = 0$$

$$r^2 + 4r + 3 = 0$$

$$\frac{-4 \pm 2}{2} \quad \frac{-2}{2} = -1, \quad \frac{-6}{2} = -3$$

$(r+1) = 0 \quad r = -1$

$\Rightarrow y_h = c_1 e^{-3x} + c_2 e^{-x} + c_3 x e^{-x}$

b) $y_p = C x^2 e^{-x} + K_2 x^2 + K_1 x + K_0 + A\cos x + B\sin x$

Q3) Use power series to solve the following D.E. at $x=0$,

$y' = 2xy$

$n = m+2$

$y = \sum_{n=0}^{\infty} a_n x^n$

$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$

$\sum_{n=1}^{\infty} a_n n x^{n-1} - \sum_{n=0}^{\infty} 2a_n x^{n+1} = 0$

$\sum_{m=-1} a_{m+2}(m+2)x^{m+1} - \sum_{m=0} 2a_m x^{m+1}$

-1

$\boxed{a_1 = 0}$

$x^{m+1}\left[a_{m+2}(m+2) - 2a_m\right] = 0$

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$a_{m+2}(m+2) = 2a_m$

$a_{m+2} = \frac{2}{m+2} a_m$

$\boxed{a_2 = a_0}$

$a_3 = \frac{2}{3} a_1 = 0 \quad a_1 = 0$

$a_4 = \frac{a_2}{2} = \frac{a_0}{2!}$

$a_5 = \frac{2}{5} a_3 = 0$

$a_6 = \frac{2}{6} a_4$

$a_6 = \frac{1}{3} a_4 = \frac{1}{3!} a_0$

$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6$

$(a_0 + a_0 x^2 + \frac{a_0 x^4}{2!} + \frac{1}{3!} x^6$

$= a_0 \left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}\right) = a_0 e^{x^2}$

Q4) Given the D.E.

$$x(x-1)y'' + (7x-1)y' + y = 0$$

a) Determine the regular points and the singular points of this equation.

b) Determine the indicial equation and its roots about $x=0$

c) Determine the recurrence relation for the coefficients a_n's. about $x=0$

a) a singular point if $x(x-1)=0$ $\quad x=0, x=1$

$(f(0), f(1))$

$y = \sum a_n x^{n+r}$, $y' = \sum a_n (n+r) x^{n+r-1}$, $y'' = \sum a_n (n+r)(n+r-1) x^{n+r-2}$

$$\sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-1} + \sum_{n=0}^{\infty} 7a_n (n+r) x^{n+r} - \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1}$$

$$+ \sum_{n=0}^{\infty} a_n x^{n+r}$$

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$n = m-1$

$$= \sum_{m=1}^{\infty} a_{m-1} (m+r-1)(m+r-2) x^{m+r-1} - \sum_{m=0}^{\infty} a_m (m+r)(m+r-1) x^{m+r-1} + \sum_{m=1} 7a_{m-1} (n+r-1) x^{m+r-1}$$

$$- \sum_{m=0}^{\infty} a_m (m+r) x^{m+r-1} + \sum_{m=1} a_{m-1} x^{m+r-1}$$

$$a_m = \frac{[(m+r-1)(m+r-2) + (n+r-1) + 1]\, a_{m-1}}{(m+r)[(m+r-1) - 1]}$$

$$a_0 (-(r)(r-1) - r) x^{r-1} =$$

$-(r)(r-1) - r$ $\quad r^2 = 0$ $\quad \boxed{r=0}$

$-r^2 + r - r = 0$

c) $x^{m+r-1} \Big[a_{m-1} (m+r-1)(m+r-2) - a_m (m+r)(m+r-1) + 7a_{m-1}(n+r-1)$

$- a_m (m+r) + a_{m-1} \Big] = 0$

$- a_m (m+r)(m+r-1) - a_m (m+r) = -a_{m-1}(m+r-1)(m+r-2) - 7a_{m-1}(n+r-1) - a_{m-1}$

$a_m (m+r)[(m+r-1) - 1] = a_{m-1}[(m+r-1)(m+r-2) + (n+r-1) + 1]$

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(Q1) **Fill in the blanks with the final answer :**
(15 points)

The characteristic equation of the D.E. $4y'' + y' + y = 0$ is $e^{-\frac{x}{8}}\left[c_1 \cos\left(\frac{\sqrt{15}}{8}x\right) + c_2 \sin\left(\frac{\sqrt{15}}{8}x\right)\right]$

(2) The Standard form of Euler – Chachy D.E. which is equivalent to $(x+1)^2 y'' - 4(x+1)y' + 6y = 0$ is ______

(3) The two functions $\sin(2x)$ and $\sin(3x)$ are linearly (independent / dependent) because independent because $W \neq 0$
$\frac{1}{2}\sin 2x \cos 3x - \frac{1}{3}\sin 3x \cos 2x \neq 0$

(4) It is known that $y_1 = \frac{\sin(x)}{\sqrt{x}}$ is a solution of $x^2 y'' + x y' + (x^2 - \frac{1}{4}) y = 0$ on $(0, \pi)$. Then a second linearly independent solution is
$y_2 = \frac{\sin x}{\sqrt{x}} \times \int \frac{x\, e^{-\frac{x^2}{2}}}{\sin^2 x}$

(5) A homogenuous second order linear D.E. with solution $y = e^{3x}(4 + 5x)$ is $y'' - 6y' + 9y = 0$

(6) The D.E. $(3x^2 y + 8xy^2)\,dx + (x^3 + 8x^2 y + 12y^2)\,dy = 0$ is (exact / not exact) because $M_y = 3x^2 + 16xy \Rightarrow N_x = 3x^2 + 16xy$ $\boxed{M_y = N_x}$

(7) If $(\cos x \sin x - x y^m)\,dx + y(1 - x^2)\,dy = 0$ is exact D.E. then m = 2

(8) The solution of the D.E. $xy' - 2y - x^5 = 0$ is $\psi(x,y) = -x^{-2}y + \frac{x^3}{3} + c$

(9) The order of the D.E. $y' = \sqrt{1 + (y'')^2}$ is Second order

(10) An integrating factor for $xy\,dx + (x^2 + 2y^2 + 2)\,dy = 0$ is y

(11) The D.E. $y' + y(1 - xy) = 0$ can be converted to a linear D.E. as $\frac{du}{dx} - u = -x$

(12) The integrating factor of $xy' - 4y = x^6 e^x$ is x^{-5}

(13) The general solution of $y' = e^{3x+2y}$ is $\psi(x,y) = \frac{e^{3x}}{3} - \frac{e^{-2y}}{2} + c$

(14) The D.E. $2x^3 y\,dx + (x^4 + y^4)\,dy = 0$ can be converted to a separable D.E. as ______

(15) The D.E. $yy'' - 2(y')^2 = 0$ can be converted to a first order D.E. as $y\frac{dw}{dy} - 2w = 0$

Q2) Solve the IVP: (4 points)

$x^2 y'' - 5xy' + 9y = 0, \quad y(1) = 2, \quad y'(1) = 0$

$y = x^m \quad , y' = m x^{m-1} \quad , y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - 5x\, m x^{m-1} + 9x^m$

$x^m [m^2 - m - 5m + 9]$

$x^m [m^2 - 6m + 9] = 0$

$m^2 - 6m + 9 = 0$

$(m-3)(m-3) = 0$

$m = 3$

$y(x) = c_1 x^3 + c_2 \ln x\, x^3$

$y'(x) = 3c_1 x^2 + \ln x\, 3c_2 x^2 + c_2 x^3 \frac{1}{x}$

$y'(1) = 3c_1 + c_2 = 0$

$y(1) = c_1 + 0 = 2$

$\boxed{c_1 = 2}$

$\boxed{c_2 = -6}$

$y(x) = 2x^3 + -6 \ln x\, x^3$

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Q3) Solve the D.E: (6 points)

$$\underbrace{(2xy^4e^y + 2xy^3 + y)}_{M}dx + \underbrace{(x^2y^4e^y - x^2y^2 - 3x)}_{N}dy = 0$$

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$M_y = 8xy^3e^y + 6xy^2 + 1$ $N_x = 2y^4e^yx - 2xy^2 - 3$

$M_y \neq N_x$ Not Exact

$$\frac{M_y - N_x}{N} = \frac{8xe^yy^3 + 6xy^2 + 1 - 2xe^yy^4 + 2xy^2 + 3}{x^2y^4e^y - x^2y^2 - 3x}$$

$$= \frac{\cancel{8xe^yy^3} - 2xe^yy^4 + 8xy^2 + 4}{x^2y^4e^y - x^2y^2 - 3x}$$

$$\frac{N_x - M_y}{M} = \frac{2y^4e^yx - 8xy^2 - 4 + 8xe^yy^3}{2xy^4e^y + 2xy^3 + y}$$

$\cancel{2xy^4e^y + 2xy^3}$

$$M(y) = e^{\int \frac{N_x - M_y}{M}}$$