University of Jordan
Electrical Engineering Department
Electromagnetics EE 253
Second Exam.
Fall 2009/2010

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Reg. #: 2076438   Section: 2 [illegible]   Time: 75 Min.

Q1 8 point	3
Q2 8 points	5
Q3 8 points	4
Q4 6 points	6
Total 30 Pts.	18

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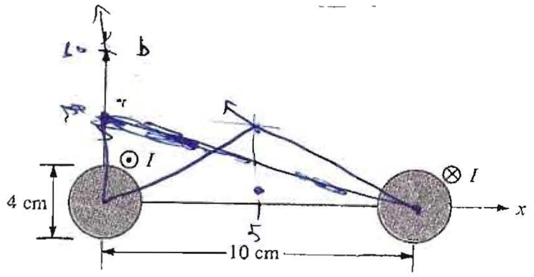
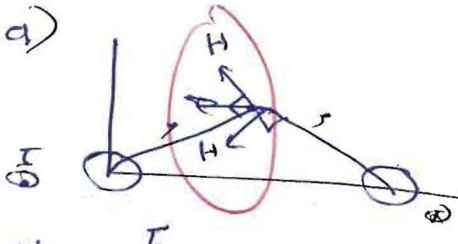
Q1. (8pts) Consider the two-wire transmission line whose cross section is illustrated in the Figure shown below. Each wire is of radius 2 cm and the wires are separated 10 cm. The wire centered at (0,0,0) carries 5 A while the other centered at (10cm,0,0) carries the return current. Find **H** at:

a) (5cm,5cm,0)
b) (0,10cm,0)

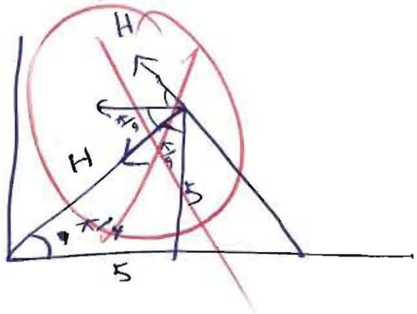


a)



$H = \frac{I}{2\pi\rho}$ $\rho = \sqrt{5^2+5^2} = 7.07$

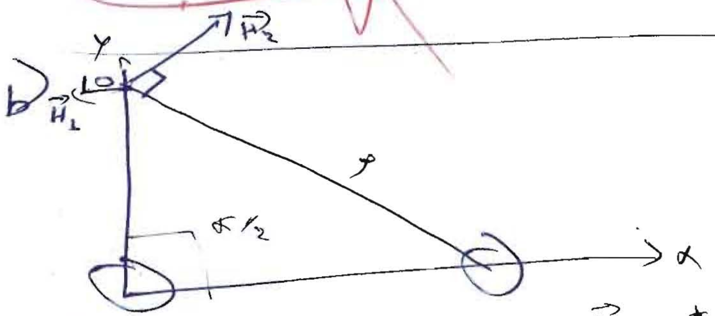
$H = \frac{5}{2\pi \times 7.07 \times 10^{-2}} = 11.253$



$\Rightarrow \vec{H} = (11.253 \cos \pi/4 + 11.253 \cos \pi/4) - \hat{a}_x$ } a_y component cancel out of summation.

$= \cancel{15.914\ \hat{a}_x}$

b)



$\vec{H}_1 = \frac{I}{2\pi\rho}(-\hat{a}_x)$

$\vec{H}_1 = \frac{5}{2\pi \times 10^{-1}}(-\hat{a}_x) = -7.9577\ \hat{a}_x$

$\Rightarrow \vec{H}_1 = +7.9577\ \hat{a}_\phi$

$\vec{H}_2 = \frac{5}{2\pi \times 14.14 \times 10^{-2}}(-\hat{a}_\phi)$ $\rho = \sqrt{10^2+10^2} = 14.14$

$= -5.626\ \hat{a}_\phi$

$\vec{H} = 2.3317\ \hat{a}_\phi$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

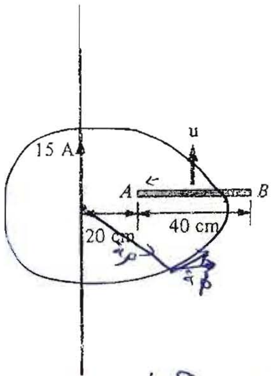
$A_\rho = 0 + 0 + 0$

$A_\phi = -a_x + 0 + 0$

$A_z = 0 + 0 + 0$

$A_\phi = -\hat{a}_x$

Q2. (8pts) A conducting rod moves with a constant speed of $5\hat{a}_z$ m/s parallel to a long straight wire carrying a 15 A current as in the Figure shown. Calculate the emf induced in the rod and state which end is at higher potential.



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$V_{emf} = \oint_L (\vec{u} \times \vec{B}) \cdot dL$

$dL = d\rho$

$\vec{u} = 5\hat{a}_z$

For infinite line

$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{15}{2\pi\rho}\hat{a}_\phi$

$\vec{B} = \frac{\mu_0 15}{2\pi\rho}\hat{a}_\phi$

$\vec{u} \times \vec{B} = (5\hat{a}_z) \times \left(\frac{\mu_0 15}{2\pi\rho}\hat{a}_\phi\right) = \frac{5 \times 15 \times \mu_0}{2\pi\rho}(-\hat{a}_\rho) = \frac{1.5\times10^{-5}}{\rho}(-\hat{a}_\rho)$

$= \int_{0.4}^{0.2} \frac{-1.5\times10^{-5}}{\rho}\hat{a}_\rho \, d\rho$

$= -1.5\times10^{-5}\hat{a}_\rho \int_{0.4}^{0.2} \frac{d\rho}{\rho}$

$= -1.5\times10^{-5} \ln\rho \Big|_{0.4}^{0.2} \hat{a}_\rho$

$= 1.0397\times10^{-5}$ V

B has higher potential.

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = 8.854 \times 10^{-12}$

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Q3.(8pts) In a certain region with $\sigma = 0$, $\mu = \mu_0$, and $\varepsilon = 6.25\varepsilon_0$, the magnetic field of the EM wave is:

$$\mathbf{H} = 0.6\cos\beta x \cos 10^8 t\, \hat{\mathbf{a}}_z \;\; A/m$$

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$\Rightarrow \vec{E} = C \cos 10^8 t \; \hat{a}_y$ both of two same length and phase shift

$E_s = C \hat{a}_y$

Find β and the corresponding $\mathbf{E}$ using Maxwell's equations.

$H_s = 0.6 \cos\beta x \, e^{j0} \, \hat{a}_z$ $\Rightarrow B_s = \mu_0 H_s$

$H_s = 0.6 \cos\beta x \, \hat{a}_z$

$= 4\pi \times 10^{-7} \times 0.6 \cos\beta x \, \hat{a}_z$

$B_s = 7.53 \times 10^{-7} \cos\beta x \, \hat{a}_z$

~~$\vec{\nabla} \times \vec{H}_s = \vec{J}$~~

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \cdot B = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ (each $= 0$)

$\vec{\nabla} \times \vec{E} = \frac{-\partial B}{\partial t}$

$\frac{\partial B_s}{\partial t} = j\omega \; 7.53 \times 10^{-7} \cos\beta x \; \hat{a}_z$

$\boxed{\vec{\nabla} \times E = +j\omega \; 7.53 \times 10^{-7} \beta \sin\beta x \; \hat{a}_z}$

$\vec{\nabla} \times E_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx} & E_{sy} & E_{sz} \end{vmatrix}$

$= \left(\frac{\partial E_{sy}}{\partial x} - \frac{\partial E_{sx}}{\partial y} \right) \hat{a}_z$

$\vec{\nabla} \times \vec{H} = \vec{J}_0 + \frac{\partial D}{\partial t}$

$\vec{J}_s = \sigma E$ ($=0$)

$+j\omega\, 0.6 \sin\beta x \; \hat{a}_y = j\omega\, \varepsilon_0 \, \vec{E}_s$

$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$

$E_s = \frac{0.6\beta \sin\beta x}{8.854 \times 10^{-12}} \; \hat{a}_y$

$\boxed{\vec{E}_s = 6.776 \times 10^{15} \beta \sin\beta x \; \hat{a}_y}$

$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{a}_x \to 0$

$\boxed{\vec{\nabla} \times E_s = j\omega\, 6.776 \times 10^{15} \beta^2 \cos\beta x \; \hat{a}_z}$

$- \left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) \hat{a}_y$

$+ \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z$ ($\to 0$)

~~$j\omega\, 6.776 \times 10^{15} \beta^2 \cos\beta x \; \hat{a}_z - +j\omega \times 7.53 \times 10^{-7} \cos\beta x \; \hat{a}_z$~~

$= -\frac{\partial H_z}{\partial x} \hat{a}_y$

$= +j\omega\, 0.6 \beta \sin\beta x \; \hat{a}_y$

$j\omega\, 6.776 \times 10^{15} \beta^2 \cos\beta x \; \hat{a}_z = j\omega\, 7.53 \times 10^{-7} \beta \sin\beta x \; \hat{a}_z$

$\boxed{\beta \cot\beta = \frac{7.53 \times 10^{-7}}{6.776 \times 10^{15}}}$

$\Rightarrow \boxed{E = 6.776 \times 10^{15} \beta \sin\beta x \cos 10^8 t \; \hat{a}_y}$

Q4. (6pts) Given $\mathbf{A} = 4sin\omega t\hat{\boldsymbol{a}}_x + 3cos\omega t\hat{\boldsymbol{a}}_y$, and $\mathbf{B}_s = j10ze^{-jz}\hat{\boldsymbol{a}}_x$, express $\mathbf{A}$ in phase form and $\mathbf{B}_s$ in instantaneous form.

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~~$A_s = 4e^{j0} + \ldots$~~

$A_s = 4e^{-j\pi/2} + 3e^{j0} \Rightarrow A_s = 4e^{-j\pi/2}\hat{a}_x + 3\hat{a}_y$

$B_s = 10e^{j\pi/2}\, z\, e^{-jz}\hat{a}_x$

$= 10z\, e^{j(\pi/2 - z)}\hat{a}_x \Rightarrow B(t, z) = 10z\cos(\omega t + \pi/2 - z)\hat{a}_x$