1-(9) 9
2-(9) 9
3-(6) 6
4-(6) 5

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Electromagnetics
EE 253
Test II
Spring 2010

Name (Arabic): ____ Reg. # ____

1- Two identical current loops have their centers at (0,0,2) and (0,0,4) and their axis the same as the z-axis. If each loop has radius 2m and carries a current 2A in the $+a_\phi$ direction. Calculate H at (0,0,0).

Loop @ (0,0,2)

$$H_1 = \frac{I a^2}{2(a^2+h^2)^{3/2}}$$

$$= \frac{\cancel{2}(4)}{\cancel{2}(4+4)^{3/2}}$$

$$= \frac{4}{8^{3/2}}$$

$$\boxed{H_1 = 0.17677}$$

loop @ (0,0,4)

$$H_2 = \frac{I a^2}{2(a^2+h^2)^{3/2}}$$

$$= \frac{\cancel{2}(4)}{\cancel{2}(4+16)^{3/2}}$$

$$= \frac{4}{20^{3/2}}$$

$$\boxed{H_2 = 0.0447}$$

$$H = H_1 + H_2$$

$$\boxed{H = 0.22147 \ A/m}$$

ok

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2- An infinitely long thin wire carries a current of 2A in the positive z-direction. Calculate the flux through the square area described by:

$2 \le \rho \le 6.0;\ 0 \le z \le 4,\ \phi = 90^\circ$

$$\Psi_m = \iint_{s} B \cdot ds$$

$$\Psi_m = \iint_{s} \mu H \cdot ds$$

$$\Psi_m = \iint_{s} \frac{\mu I}{2\pi\rho} \cdot ds$$

$$\Psi_m = \iint_{s} \frac{\mu I}{2\pi\rho}\, d\rho\, dz\ a_\phi$$

$$\Psi_m = \int_{\rho=2}^{\rho=6}\int_{0}^{4} \frac{\mu I}{2\pi\rho}\, d\rho\, dz$$

$$= \int_{\rho=2}^{\rho=6} \left[\frac{z\mu I}{2\pi\rho}\right]_0^4 d\rho$$

$$= \frac{4\mu I}{2\pi} \ln\left(\frac{6}{2}\right)$$

$$\Psi_m = \frac{4(\cancel{4}^{2}\cancel{\pi}\, 10^{-7})(2)}{\cancel{2}\cancel{\pi}} \ln(3)$$

$$\Psi_m = 16(10^{-7}) \ln(3)$$

$$\boxed{\Psi_m = 175.78\ \mu Wb}$$

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3- If **H** = y$\mathbf{a}_x$ −x$\mathbf{a}_y$ A/m on plane z=0, determine the current density.

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

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$$= \frac{\partial}{\partial z}(x)\, a_x + \frac{\partial}{\partial z}(y)\, a_y + \left[\frac{\partial}{\partial x}(-x) - \frac{\partial}{\partial y}(y)\right] a_z$$

$$= 0\, a_x + 0\, a_y + \left[(-1) - 1\right] a_z$$

$$\boxed{J = -2\, a_z}$$

4- State Maxwell's four equations and state each equation was derived from which law.

1. $\vec{\nabla} . \vec{D} = \rho_v \longrightarrow$ derived from Gaussian law
2. $\vec{\nabla} . \vec{B} = 0 \longrightarrow$ derived from Gaussian law
3. $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t} \longrightarrow$ derived from Stokes Theorem
4. $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \longrightarrow$ derived from Ampere's law

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① $\vec{\nabla} . \vec{D} = \rho_v$

derived from $\boxed{\text{Gaussian law}}$

$$\oint\!\!\!\oint_{s} D . ds = \iiint \rho_v \, dv$$

② $\vec{\nabla} . \vec{B} = 0$

derived from $\boxed{\text{Gaussian law}}$

$$\oint\!\!\!\oint_{s} B . ds = 0$$

③ $\vec{\nabla} \times \vec{E} = \frac{-\partial B}{\partial t}$

derived from $\boxed{\text{Stokes Theorem}}$

$$\oint B . dl = \iint (\vec{\nabla} \times B) . ds$$

④ $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$

$\vec{\nabla} \times H = J$

derived from $\boxed{\text{Ampere's law}}$

$$\oint H . dL = \iint_{s} J . ds$$