Probability & Random Variables

chap 3 + Quizes

EE321 Quize#1 time:40 mins

Q1- A car arriving at a crossroads is equally likely to turn left (L), to turn right (R), or to carry straight on (S).
(a) What is the probability that it makes a turn? Given that it makes a turn, what is the probability that it turns left?
(b) Two cars both arrive at the crossroads, and each behaves independently as above. What is the probability that at least one car turns left? Given that at least one car makes a turn, what is the probability that at least one car turns left?

3. (a)

$$\begin{aligned} \text{P(car makes a turn)} &= \text{P}(L \cup R) = \text{P}(L) - \text{P}(R) \quad \text{(since disjoint)} \\ &= \tfrac{1}{3} + \tfrac{1}{3} = \boxed{\tfrac{2}{3}} \end{aligned}$$

$$\begin{aligned} \text{P}(L|\text{makes a turn}) &= \frac{\text{P(makes a turn}|L)\text{P}(L)}{\text{P(makes a turn)}} \\ &= \frac{1 \times \frac{1}{3}}{\frac{2}{3}} = \boxed{\tfrac{1}{2}} \end{aligned}$$

(b) Let C_i = event that car i makes a turn, $i = 1, 2$.

$$\begin{aligned} \text{P(at least one car makes a turn)} &= 1 - \text{P(neither car makes a turn)} \\ &= 1 - \text{P}(C_1' \cap C_2') \\ &= 1 - \text{P}(C_1')\text{P}(C_2') \quad \text{(independent)} \\ &= 1 - \tfrac{1}{3} \times \tfrac{1}{3} = \boxed{\tfrac{8}{9}} \end{aligned}$$

$$\begin{aligned} \text{P(at least one car makes a turn)} &= \text{P(car 1 makes a turn)} + \\ &\quad \text{P(car 2 makes a turn)} - \text{P(both cars make a turn)} \\ &= \text{P}(C_1) + \text{P}(C_2) - \text{P}(C_1 \cap C_2) \\ &= \text{P}(C_1) + \text{P}(C_2) - \text{P}(C_1) \times \text{P}(C_2) \quad \text{(independent)} \\ &= \tfrac{2}{3} + \tfrac{2}{3} - \left(\tfrac{2}{3}\right)^2 = \boxed{\tfrac{8}{9}} \end{aligned}$$

$$\begin{aligned} \text{P(at least one car turns left)} &= 1 - \text{P(neither car turns left)} \\ &= 1 - \left(\tfrac{2}{3}\right)^2 = \boxed{\tfrac{5}{9}} \end{aligned}$$

Let A = event that at least one car makes a turn
Let B = event that at least one car turns left

$$\begin{aligned} \text{P}(B|A) &= \frac{\text{P}(A|B)\text{P}(B)}{\text{P}(A)} \\ &= \frac{1 \times \frac{5}{9}}{\frac{8}{9}} = \boxed{\tfrac{5}{8}} \end{aligned}$$

EE321 Quize#1 time:40 mins

Q1- A company makes resistors which are nominally 15 ohms. In fact 5% of the output are below 14 ohms and 10% are above 16 ohms. Two resistors are selected at random.
(a) What is the probability that both are between 14 and 16 ohms?
(b) What is the probability that at least one is above 16 ohms?

$P\{R < 14\} = 0.05$

$P\{R > 16\} = 0.1$

$P\{14 < R_1 \leq 16\} \cap \{14 \leq R_2 \leq 16\} =$

$P\{14 \leq R_1 \leq 16\} \cdot P\{14 \leq R_2 \leq 16\}$

Now $P\{14 < R \leq 16\} = 1 - P\{R \leq 14\} \cup \{R > 16\}$

$= 1 - [P\{R \leq 14\} + P\{R > 16\}$

$= 1 - [.05 + 0.1] = 0.85$

a) $\Rightarrow (0.85)^2 = \boxed{0.7225}$

b) $P\{$at least one is above 16$\} = 1 - P\{$both $\leq 16\}$

$= 1 - P\{R_1 \leq 16\} \cap \{R_2 \leq 16\}$

$= 1 - (0.9)^2 = \boxed{0.19}$

Q2- . Suppose that telephone calls have random lengths (in minutes) with exponential density

$$f_X(x) = \frac{1}{3}\exp\left(-\frac{x}{3}\right) \quad x > 0.$$

Let X denote the length of a call.
(a) Find P(X < 2).
(b) Find the probability that a call lasts between 4 and 7 minutes
(c) Find the probability that a call exceeds 4 minutes.
(d) What is the conditional probability that a call lasts less than 7 minutes, given that it has already lasted 4 minutes?

3.

$$\begin{aligned} P(X < x) &= F_X(x) = \int_0^x f_X(t)\,dt \\ &= \int_0^x \frac{1}{3}e^{-\frac{t}{3}}\,dt \\ &= \left[-e^{-\frac{t}{3}}\right]_0^x \\ &= 1 - e^{-\frac{x}{3}} \end{aligned}$$

(a) $P(X < 2) = F_X(2) = 1 - e^{-\frac{2}{3}} = \boxed{0.487}$.

(b) $P(4 \le X \le 7) = F_X(7) - F_X(4) = e^{-\frac{4}{3}} - e^{-\frac{7}{3}} = \boxed{0.167}$.

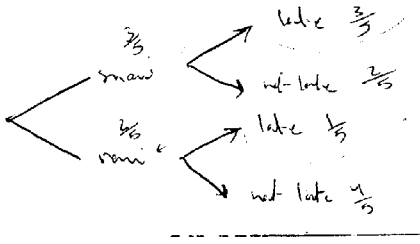
(c) $P(X > 4) = 1 - F_X(4) = e^{-\frac{4}{3}} = \boxed{0.264}$.

(d)

$$\begin{aligned} P(X < 7 | X > 4) &= \frac{P(X < 7 \cap X > 4)}{P(X > 4)} \\ &= \frac{P(4 < X < 7)}{P(X > 4)} \\ &= \frac{e^{-\frac{4}{3}} - e^{-\frac{7}{3}}}{e^{-\frac{4}{3}}} \\ &= \frac{e^{-\frac{4}{3}}(1 - e^{-1})}{e^{-\frac{4}{3}}} \\ &= 1 - e^{-1} = \boxed{0.632} \end{aligned}$$

EE321 Quize#1 time:40 mins

Q1- Tomorrow there will be either rain or snow but not both; the probability of rain is 2/ 5 and the probability of snow is 3 /5 . If it rains then the probability that I will be late for my lecture is 1/ 5 , while the corresponding probability in the event of snow is 3/5 . What is the probability that I will be late?



P(late/snow) + P(late/rain)

$\frac{3}{5} \times \frac{3}{5}$ + $\frac{1}{5} \times \frac{2}{5}$

$\frac{9}{25} + \frac{2}{25} = \frac{11}{25}$

1. Let S = event that it snows tomorrow,
   R = event that it rains tomorrow,
   L = event that I am late.

$$P(S) = \frac{3}{5}, \quad P(R) = \frac{2}{5}, \quad P(L|R) = \frac{1}{5}, \quad P(L|S) = \frac{3}{5}.$$

$$\begin{aligned} P(L) &= P(L|R)P(R) + P(L|S)P(S) \\ &= \tfrac{1}{5} \times \tfrac{2}{5} + \tfrac{3}{5} \times \tfrac{3}{5} = \boxed{\tfrac{11}{25}} \end{aligned}$$

1/10/2006

EE321 QUIZE#1 TIME: 10 MINTS

Q1- A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease.
Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.
Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

EE321 QUIZE#1 TIME: 10 MINTS

Q2- A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:
(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third have high blood pressure.
(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.
What portion of the patients selected have a regular heartbeat and low blood pressure?