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EE321 | **First Exam** | **Time: 1 Hours**

Q1: A uniform random voltage X can have $-6 \le x \le 6$, a quantizer divides X in 5 subsets and generate random variable Y having values as giving in the following table

$x_i \le X \le x_{i+1}$	$-6 < x \le -3$	$-3 < x \le -1$	$-1 < x \le 1$	$1 < x \le 3$	$3 < x \le 6$
Y=yi	-4	-2	0	2	4

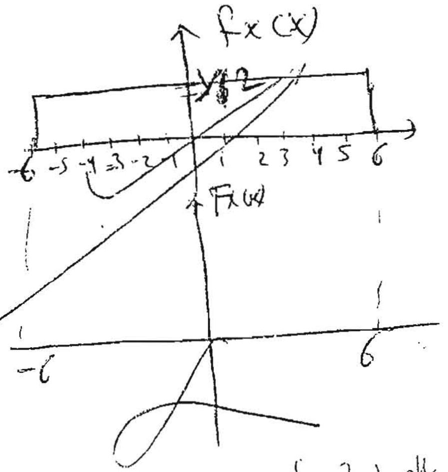
a) sketch $f_x(x)$, $F_X(x)$

b) find and sketch $f_y(y)$

c) find the following probabilities $P\{Y \le 1.5\}, P\{Y \le 10\}, P\{Y < -6\}$

d) Find $E[Y], \sigma_Y^2, \sigma_x^2$

$f_X(x) = \begin{cases} 1/12, & -6 \le x < 6 \\ 0, & \text{o.w} \end{cases}$

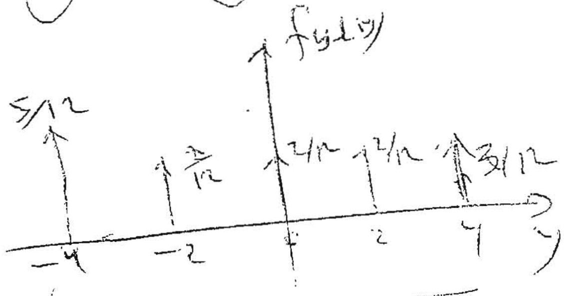


$F_X(x) = \int_{-\infty}^{x} f_X(\lambda)\, d\lambda = \int_{-6}^{x} \frac{1}{6}\, d\lambda = \frac{1}{6}\lambda \Big|_{-6}^{x} = \frac{1}{6}(x+6)$

So $F_X(x) = \begin{cases} 1, & x > 6 \\ \frac{x+6}{6}, & -6 < x < 6 \\ 0, & x < -6 \end{cases}$

$\int x^2 \frac{1}{12}\, dx$

b) $f_y(y) = \frac{5}{12}\delta(y+4) + \frac{2}{12}\delta(y+2) + \frac{2}{12}\delta(y) + \frac{2}{12}\delta(y-2) + \frac{3}{12}\delta(y-4)$



c) $p\{y \le 1.5\} = p\{y=0\} + p\{y=-2\} + p\{y=-4\}$

$= \frac{2}{12} + \frac{2}{12} + \frac{5}{12} = \frac{9}{12}$

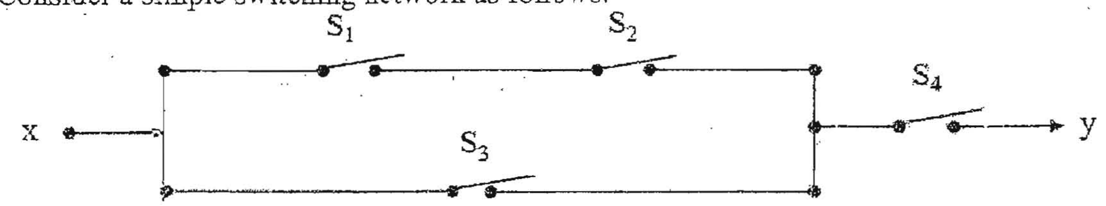
$p\{y \le 10\} = 1$

$p\{y < -6\} = 0$

$\sigma_x^2 = \dots = E[x^2] + \bar{x}^2$

$6x = 12$

Q2: A) Consider a simple switching network as follows:



Define: Ak = {switch Sk fails}, k = 1,2,3,4 ,Pr[Ak] = p
Find the probability that the path between x and y is established.

$p((\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_4) \cup (\bar{A}_3 \cap \bar{A}_4))$

$= p(\bar{A}_1 \bar{A}_2 \bar{A}_4) + p(\bar{A}_3 \cap \bar{A}_4) - p(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4)$

$= (1-p)(1-p)(1-p) + (1-p)(1-p) - (1-p)^4$

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B) Consider a sequence of 3 binary numbers (occurring randomly). What is the probability that there are more 1's than 0's given that the first bit (from the left) is a 1.

p(more 1's than 0's | first bit is 1) = p(more 1's and 0's ∩ first bit 1

0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

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p(more 1's than 0's given) = $\frac{2}{3}$

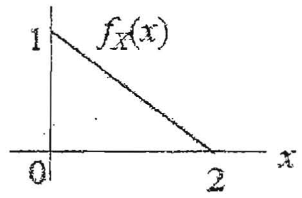
Q3: The PDF of a random variable X is given by the figure shown

a) Write an expression of $f_x(x)$, $F_X(x)$

b) Find the mean, the second moment, and the variance of X

c) Now consider a random variable $Y = 2X + 3$, find $f_y(y)$

d) Determine the mean and the variance of $2X + 3$.



$(2,0)$
$(0,1)$

$m = \frac{0-1}{2} = \frac{-1}{2}$

$y - 0 = \frac{-1}{2}(x-2)$

$f_x(x) = y = -\frac{1}{2}(x-2)$

a) $f_x(x) = \begin{cases} \frac{-1}{2}(x-2) , & 0 \le x \le 2 \\ 0 , & \text{o.w} \end{cases}$

$F_x(x) = \int_{-\infty}^{x} f_x(\tau)\, d\tau = \int_{0}^{x} -\frac{1}{2}(\tau - 2)\, d\tau$

$= \frac{-1}{2}\left[\left(\frac{\tau^2}{2} - 2\tau\right)\Big|_0^x\right]$

$= \frac{-1}{2}\left[\frac{x^2}{2} - 2x\right] , \quad 0 \le x < 2$

$= \frac{-x^2}{4} + x$

~~$F_x(x) = \left\{ \frac{-x^2}{4} + x , \quad 0 < x < 2 \right.$~~

$F_x(x) = \begin{cases} 1 , & x > 2 \\ \frac{-x^2}{4} + x , & 0 < x < 2 \\ 0 , & x < 0 \end{cases}$

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اذا عمل $y = 2x+3$ ...

b) $\bar{X} = E[X] = \int_{-\infty}^{\infty} x f_x(x)\, dx = \int_{0}^{2} (x)\left(-\frac{1}{2}(x-2)\right) dx$

$= -\frac{1}{2}\int_{0}^{2} x^2 - 2x\, dx$

$= \frac{-1}{2}\left[\left(\frac{x^3}{3} - \frac{2x^2}{2}\right)\Big|_0^2\right]$ ... تكملة الحل

Q4: A certain large city averages three murders per week and their occurrences follow a Poisson distribution.

(a) What is the probability that there will be five or more murders in a given week?
(b) On the average, how many weeks a year can this city expect to have no murders?
(c) How many weeks per year (average) can the city expect the number of murders per week to equal or exceed the average number per week?

For Poisson RV

$$f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)$$

$\lambda = 3$ murders per week

$b = 3$

a) $P\{\text{will be five or more murders}\} = P\{k \geq 5\} = 1 - P\{k < 5\}$

$= 1 - \{P(k=0) + P\{k=1\} + P\{k=2\} + P\{k=3\} + P\{k=4\}\}$

$= 1 - \left(e^{-3} \frac{3^0}{0!} + e^{-3} \frac{3^1}{1!} + e^{-3} \frac{3^2}{2!} + e^{-3} \frac{3^3}{3!} + e^{-3} \frac{3^4}{4!} \right)$

$= 1 - e^{-3} (1 + 3 + 4.5 + 4.5 + 3.375)$

$= .1847$

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b) $P\{k=0\}$ $\quad b = \lambda T = (3)(52) = 156$

$= e^{-156} \left(\frac{156^0}{0!} \right) = 3$

$P\{x=0\}$
$52 \times .0498$
$= 2.59$
week

c)

$P\{\text{murders} \geq \text{avg}\} = P\{ > \ldots \}$

$P\{X \geq 3\}$ $\quad = 1 - P\{X < 3\}$

$1 - P\{x < 3\}$

$= \ldots$ week

$\left(1 - e^{-3} \left(\frac{3^2}{52!} \right) \right)$