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22/30

Jordan University
Faculty of Engineering and Technology
Electrical Engineering Department
Fall 2005

EE 321 | Mid Exam | Time: 90 minutes

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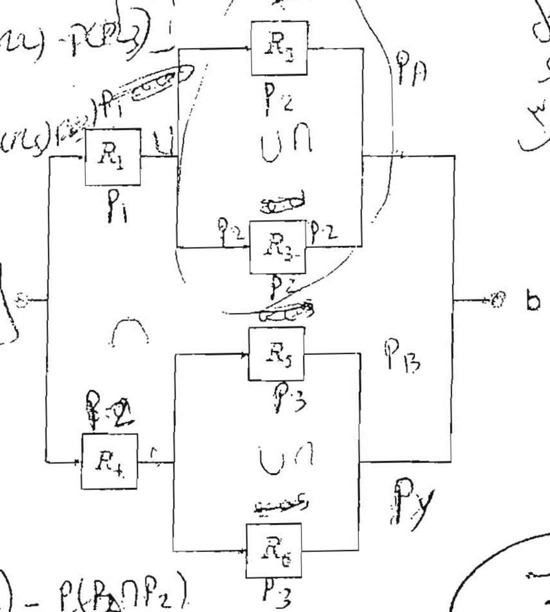
Q1- In a communication system the signal sent from point (a) to point (b) passing a series repeaters shown in the following figure. The probabilities of the repeaters failing (independently) are p1 = P[R1] = 0.005, p2 = P[R2] = P[R3] = P[R4] = 0.01, and p3 = P[R5] =P[R6] =0.05. Find the probability that the signal will not arrive at point b. (7 marks)



$P\{R_1 \cup (R_2 \cap R_3)\} + P\{R_4 \cup R_5 \cap R_6\}$

$[P(R_1) + P(R_2)P(R_3) - P(R_1)P(R_2)P(R_3)] \times P(R_4) + P(R_5)P(R_6) - P(R_4)P(R_5)P(R_6)$

$= .005 + (.01)^2 - (.005)(.01)^2 \cdot .01 + (.05)^2 - .01 \times (.05)^2$

$= .000064$

$P = P_1P_2$

$P_A = P_2^2$

$P_B = P_3^2$

7

$P_A\{P_2 \cup P_2\} = P(P_2) + P(P_2) - P(P_2 \cap P_2)$

$= 0.01 + 0.01 - (0.01 * 0.01) = 0.0199$

$P_B\{P_3 \cup P_3\} = P(P_3) + P(P_3) - P(P_3)P(P_3) = 0.05 + 0.05 - (0.05)^2 = 0.0975$

$P(R_1 \cap P_A) = P(P_1) * P(P_A) = 0.005 * 0.0199 = 0.0001 = P_x$

$P(R_4 \cap P_B) = P(R_4) * P(P_B) = 0.01 * 0.0975 = 0.000975 = P_y$

$P\{x \cup y\} = P\{x\} + P\{y\} - P\{x \cap y\}$

$P\{b\} = P_x + P_y - (P_x * P_y) = 0.001075 - (9.75 * 10^{-8})$

$= 0.001075$

Q2- Earthquakes occur in a certain country in a Poisson process of rate 0.4 per year.
(a) Determine the probability of one earthquake in three years.
(b) Determine the probability of no earthquakes in three years.
(c) What is the probability of at most two earthquakes in one year?
(d) What is the probability of at least one earthquake in five years?

The pdf $$f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)$$

$b = 0.4$ $$F_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} u(x-k)$$ $\lambda = 0.4$

(a) $$e^{-0.4}\left(\frac{0.4}{1!} + \frac{(0.4)^2}{2!} + \frac{(0.4)^3}{3!}\right) = 0.49\, e^{-0.4}$$

$\cancel{e^{-0.4}}$

$b = 1.2$

P(one earthquake)

$P(k=1)$

$e^{-1.2} \frac{b^k}{k!}$

(b) $e^{-0.4}\ ($

2

(c) $P\{x<2\}$

$$e^{-0.4}\left(\frac{0.4^1}{1!} + \frac{0.4^2}{2!}\right)$$

its value

$$\frac{0.4}{1} e^{-0.4} u(x-1) + \frac{0.4^2}{2} e^{-0.4} u(x-2)$$

(d) $P\{x>1\}$

$$\frac{0.4^1}{1} e^{-0.4} u(x-1) + \frac{(0.4)^2}{2!} u(x-2) + \frac{(0.4)^3}{3!} e^{-0.4} u(x-3) + e^{-0.4} \frac{(0.4)^4}{4!} u(x-4)$$

$$+\ e^{-0.4} \frac{(0.4)^5}{5!} u(x-5)$$

Value

Q2

Suppose you have two coins, one biased, one fair, but you don't know which coin is which. Coin 1 is biased. It comes up heads with probability 3/4, while coin 2 will flip heads with probability 1/2. Suppose you pick a coin at random and flip it. Let C_i denote the event that coin i is picked. Let H and T denote the possible outcomes of the flip. Given that the outcome of the flip is a head, what is $P[C_1|H]$, the probability that you picked the biased coin? Given that the outcome is a tail, what is the probability $P[C_1|T]$ that you picked the biased coin?

coin 1 (biased)

$p(H) = 3/4 \Rightarrow p(T) = \frac{1}{4}$

coin 2 (fair)

$p(H) = \frac{1}{2}, \; p(T) = \frac{1}{2}$

~~$p(C_1$~~

$p(H) = p(H \backslash C_1) * p(C_1) + p(H \backslash C_2) * p(C_2)$

1- $p(C_1 \backslash H) = \frac{p(H \backslash C_1) * p(C_1)}{p(H)}$

$= \frac{\frac{3}{4} * \frac{1}{2}}{\frac{3}{4} * \frac{1}{2} + \frac{1}{2} * \frac{1}{2}} = 0.6$

7.5

2- $p(C_1 \backslash T) = \frac{p(T \backslash C_1) * p(C_1)}{\cancel{p(T \backslash C_1) * p(C_1)} \; p(T)}$

$p(T) = p(T \backslash C_1) * p(C_1) + p(T \backslash C_2) * p(C_2)$

$= \frac{1}{4} * \frac{1}{2} \rightarrow \frac{1}{2} * \frac{1}{2} = 0.375$

$p(C_1 \backslash T) = \frac{\frac{1}{4} * \frac{1}{2}}{.375} = \frac{1}{3}$