مبادئ الاحصاء-- ف1-2009/2008--الامتحان الثاني (لن تقبل أية اجابه صحيحه بدون تفاصيل الحل)

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Q1) Let A and B be mutually exclusive (*disjoint*) events such that $P(A) = 0.6$. Find $P(B|A)$.

$P(B|A) = \frac{P(A \cap B)}{P(B)} = \frac{0}{\cdot} = 0$

Q2) Let A, B be independent events such that $P(A) > 0$ and $P(A \cap B) = 3P(A \cap \bar{B})$. Find $P(B)$.

$P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A) - 3P(A \cap \bar{B})$

$\Rightarrow 4P(A \cap \bar{B}) = P(A)$

$\hookrightarrow 4P(A)P(\bar{B}) = P(A) \Rightarrow P(\bar{B}) = 0.25$

Q3) There are two boxes. Box1 contains 1 black and 5 white balls. Box2 contains 2 black and 4 white balls. One ball is randomly selected from one of the two boxes

a) find the probability that the ball is white

$\frac{1}{2} \cdot \frac{5}{6} + \frac{1}{2}\left(\frac{4}{6}\right)$ → $\frac{9}{12}$

b) if the selected ball is found to be white, what is the probability that it came from box1.

$P(\text{box}_1 \mid W) = \frac{P(W \mid \text{box}_1)\,P(\text{box}_1)}{P(W)} = \frac{\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)}{\frac{9}{12}}$ → $\frac{5}{9}$

Q4) For the following distribution of a random variable X, find

k	$P(X=k)$
−2	0.1
−1	0.3
0	0.3
1	0.2
2	0.1

a) $P(X > -1 \mid X \text{ is even}) = \frac{0.4}{0.5}$ → 0.8

b) $E(X^2)$ → 1.3

x	$x^2 \cdot p(x)$
−2	0.4
−1	0.3
0	0
1	0.2
2	0.4
	1.3

Q5) For the following (bivariate) distribution

X \ Y	−2	0	2	
−1	0.2	0.2	0.2	0.6
1	0.1	0.2	0.1	0.4
	0.3	0.4	0.3	

a) find $Corr(X,Y)$. → 0

$corr = \frac{cov(x,y)}{\sigma_x \sigma_y}$

$cov(x,y) = E(xy) - E(x)E(y) = 0 - (-0.2)(0) = 0$

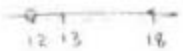
b) are X and Y independent? Why / why not?

No, because $P(x_i, y_j) \neq P(x = x_i)\,P(y = y_j)$ for all values of x, y

Q6) Let $X \sim B(20, 0.7)$. Find

a) $E(X - Var(X))$ 9.8

b) $P(12 < X \leq 18) = P(X \leq 18) - P(12 \leq X)$ 0.764



Q7) The weights of six-year-old children is normally distributed with mean $\mu = 20$ kg and standard deviation $\sigma = 6$ kg. Find $X \sim N(20, 6^2)$

a) the percentage of six-year-old children whose weights are between 8 kg and 23 kg

$P(8 < X < 23) = P(-2 < Z < 0.5)$ 0.6687

$= P(Z < 0.5) - P(Z < -2) =$

b) the 90th percentile P_{90} of the weights of six-year-old children 27.74

$P\left(Z < \frac{P_{90} - \mu}{\sigma}\right) = 0.9$

$\Rightarrow \frac{P_{90} - 20}{6} = 1.29$

Q8) A fair coin (قطعة نقد معدنية متناظرة) is tossed (رميت) 100 times. Let X be the number of times a head will come out. Use the normal approximation to approximate $P(X < 55)$. 0.8159

$X \sim B(100, 0.5) \Rightarrow$ عند $X \sim N(50, 25)$

$P(X < 55) = P(X \leq 54) \approx P(X < 54.5)$

$P(Z < 0.9)$

Q9) The number of typos (أخطاء مطبعية) is distributed according to a Poisson distribution with mean 2 typos per page. Let X be the number of typos in a randomly selected sample of 5 pages. $X \sim$ Poisson (10)

Find a) $E(X) = 2 \cdot 5$ 10

b) $P(10 < X \leq 14)$ 0.334

$P(X \leq 14) - P(X \leq 10)$

$0.917 - 0.583$